

Sample Question Sheet 1

No calculators allowed:

1) Convert these numbers to or from scientific notation:

- a) 0.0001478 b) 1483700 c) 5.24578×10^4 d) 6.23×10^5 e) 4.125×10^{-3}
 1.478×10^{-4} 1.4837×10^6 52457.8 623000 0.004125

2) Perform the following calculations rounding your answers to the appropriate number of significant figures:

a) $8.001 + 0.2456 + 103.1$

$101.3466 \approx 101.3$ (4 s.f.)

b) $(2.34 \times 10^2) \times 1.10$

$$\begin{array}{r} 234 \\ \times 1.1 \\ \hline 234 \\ 2340 \\ \hline 257.4 \end{array} \approx 257$$
 (3 s.f.)

3) Simplify the following:

a) $\sqrt[3]{12} + 3\sqrt[3]{6} + 4\sqrt[3]{3} - \sqrt[3]{6}$
 $2\sqrt[3]{3} + 2\sqrt[3]{6} + 4\sqrt[3]{3} = 6\sqrt[3]{3} + 2\sqrt[3]{6}$

b) $3\sqrt{18} + 5\sqrt[3]{72} - 2\sqrt[3]{50}$
 $= 9\sqrt{2} + 5 \times 6\sqrt[3]{2} - 2 \times 5\sqrt[3]{2}$
 $= (9 + 30 - 10)\sqrt[3]{2} = 29\sqrt[3]{2}$

4) Simplify the following:

a) $2^3 \times 4^3$
 $= 2^3 \times (2^2)^3 = 2^3 \times 2^6 = 2^9$

b) $4^4 \times 4^3 \times 4^{-8}$
 $4^{(4+3-8)} = 4^{-1} = \frac{1}{4}$

c) $\frac{6^5 \times 3^5}{2^5}$
 $= \frac{2^5 \times 3^5 \times 3^5}{2^5} = 3^{10}$

d) $\frac{5^{-3} \times 5^6}{5^{-7} \times 5^4}$
 $= 5^{-3+6+7-4} = 5^6$

5) Simplify the following:

a) $4\sqrt{2} \times 3\sqrt{5} \times 5\sqrt{10}$
 $60\sqrt{100} = 600$

b) $3\sqrt[3]{81} \times 4\sqrt[3]{3}$
 $= 3^3\sqrt[3]{27} \times 4\sqrt[3]{3} = 3 \times 3 \times 4 \times \sqrt[3]{9} = 36\sqrt[3]{9}$

c) $\frac{-10\sqrt{80}}{15\sqrt{5}}$
 $= \frac{-10 \times 4\sqrt{5}}{15\sqrt{5}} = \frac{-40}{15} = \frac{-8}{3}$

6) Expand and simplify:

a) $(\sqrt{5} - 4)^2$
 $= 5 - 2 \times 4\sqrt{5} + 16 = 21 - 8\sqrt{5}$

b) $(\sqrt{20} - 3)(2 + \sqrt{20})$
 $= 20 - 6 - 3\sqrt{20} + 2\sqrt{20} = 14 - \sqrt{20} = 14 - 2\sqrt{5}$

7) Express the following fractions with a rational denominator:

a) $\frac{\sqrt{3}}{6\sqrt{6}} = \frac{\sqrt{3}\sqrt{6}}{6\sqrt{6}\sqrt{6}} = \frac{3\sqrt{2}}{36} = \frac{\sqrt{2}}{12}$

b) $\frac{3}{\sqrt{10+1}} = \frac{3(\sqrt{10}-1)}{(\sqrt{10+1})(\sqrt{10}-1)} = \frac{3(\sqrt{10}-1)}{9} = \frac{1}{3}(\sqrt{10}-1)$

9) Convert the following repeating decimals to fractions:

a) $0.132132132132\dots$

$x = 0.1\bar{3}2$
 $1000x = 132.1\bar{3}2$
 $\therefore 999x = 132$
 $\therefore x = \frac{132}{999}$

b) $8.0474747\dots$

$x = 8.0\bar{4}7$
 $1000x = 8047.\bar{4}7$
 $10x = 80.\bar{4}7$
 $\therefore 990x = 7967$
 $\therefore x = \frac{7967}{990}$

$y = 0.0\bar{4}7$
 $10y = 0.\bar{4}7$
 $1000y = 47.\bar{4}7$
 $990y = 47$
 $\therefore y = \frac{47}{990}$
 $\therefore x = 8\frac{47}{990}$

c) $9.1744444\dots$

$x = 9.174\bar{4}4$
 $y = 0.174\bar{4}4$
 $1000y = 174.\bar{4}44$
 $10000y = 1744.\bar{4}44$
 $\therefore 9000y = 157$
 $\therefore y = \frac{157}{900}$
 $\therefore x = 9\frac{157}{900}$

10) Solve for x:

a) $3x^2 + 108 = 0$ $x^2 = -36$
 $\therefore x = \pm 6i$

11) Simplify:

a) $\frac{1+i}{2-i}$
 $\frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{2-1+3i}{2+1} = \frac{1+3i}{3}$

b) $2i + \frac{3-i}{1+3i} = x$
 $\frac{(3-i)(1-3i)}{(1+3i)(1-3i)} = \frac{3-3-10i}{10} = -i$
 $\therefore x = i$

c) $(\frac{1}{2} + \frac{\sqrt{3}}{2}i)^3 + i^2$
 $\frac{1}{8}(1+\sqrt{3}i)(1-\sqrt{3}i+2\sqrt{3}i)$
 $= \frac{2}{8}(1+\sqrt{3}i)(-1+\sqrt{3}i) + i^2$
 $= \frac{2}{8}(-1-3) + i^2 = \frac{-8}{8} - 1 = -2$

12) Simplify:

a) $\frac{3^{k+1} \times 9^k \times 2^5}{27^k \times 4^2}$
 $= 3^{k+1} \times 3^{2k} \times 2^5 \times 3^{-3k} \times 2^{-4}$
 $= 2 \times 3^{3k+1-3k} \times 3$
 $= 2 \times 3$
 $= 6$

b) $\frac{3\sqrt{8}}{\sqrt{2}-1} + \frac{\sqrt{3}+1}{\sqrt{6}-2}$
 $= \frac{3\sqrt{8}(\sqrt{2}+1)}{1} + \frac{(\sqrt{3}+1)(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)}$
 $= 6\sqrt{2}(\sqrt{2}+1) + \frac{\sqrt{18} + \sqrt{6} + 2\sqrt{3} + 2}{2}$
 $= 12 + 6\sqrt{2} + \frac{3\sqrt{2}}{2} + \frac{\sqrt{6}}{2} + \sqrt{3} + 1 = 13 + \frac{13\sqrt{2}}{2} + \frac{\sqrt{6}}{2} + \sqrt{3}$

c) $\frac{a^2 \sqrt[3]{a^4 b^2}}{a^3 \sqrt[3]{a^7 b^4}}$
 $= a^2 a^{4/3} b^{2/3} a^{-3} a^{-7/3} b^{-4/3}$
 $= a^{-2} b^{-2/3}$
 $= \frac{1}{a^2 \sqrt[3]{b^2}}$

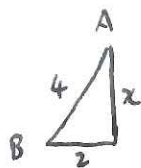
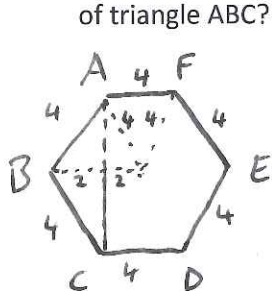
13) Suppose $x = 53^{67} + 53^{-67}$ and $y = 53^{67} - 53^{-67}$. What is $x^2 - y^2$?

$x^2 - y^2 = (x+y)(x-y) = (53^{67} + 53^{-67} + 53^{67} - 53^{-67})(53^{67} + 53^{-67} - 53^{67} + 53^{-67})$
 $= (2 \times 53^{67})(2 \times 53^{-67})$
 $= 4$

14) Simplify the following:

a) $\frac{x + 2\sqrt{xy} + y}{\sqrt{x} + \sqrt{y}} = \frac{(\sqrt{x} + \sqrt{y})(\sqrt{x} + \sqrt{y})}{(\sqrt{x} + \sqrt{y})} = \sqrt{x} + \sqrt{y}$

15) Suppose ABCDEF is a regular hexagon, labeled clockwise, with sides of length 4. What is the area of triangle ABC?



$x^2 = 4^2 + 2^2 = 20$
 $\therefore x = \sqrt{20} = 2\sqrt{5}$
 $\therefore AC = 2 \times 2\sqrt{5} = 4\sqrt{5}$
 $\therefore \Delta ABC = \frac{1}{2} \times 4\sqrt{5} \times 2 = 4\sqrt{5}$