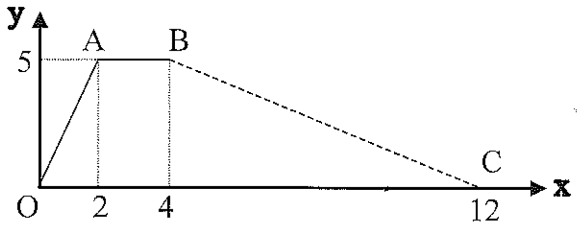
Sample Question Sheet 3

No calculators allowed.

1) Write down the equations of the 4 lines forming the sides of the trapezium OABC as shown:



$y=5$

$y=0$

$2y=5x$

$y=-\frac{5x}{8}+c$

$(12,0) \Rightarrow 0=-\frac{15}{2}+c \therefore c=\frac{15}{2}$

$y=-\frac{5x}{8}+\frac{15}{2}$ or $8y+5x=60$

Now write out 4 inequalities to define the area within the trapezium.

$y \leq 5$

$y \geq 0$

$2y \leq 5x$

$8y+5x<60$

Now find the integer coordinates which maximise the following values:

i) $x+y$ $(10,1) \Rightarrow 11$ and $(11,0) \Rightarrow 11$

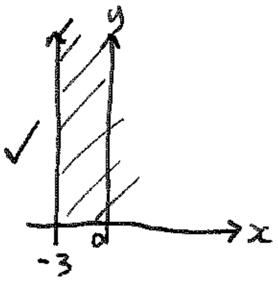
ii) $y-x$ $(5,2) \Rightarrow 3$

iii) $x-y$ $(11,0) \Rightarrow 11$

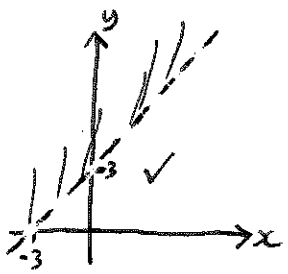
iv) ~~$y-2x$~~ $x+2y$ $(3,5) \Rightarrow 13$

2) Sketch the following inequalities:

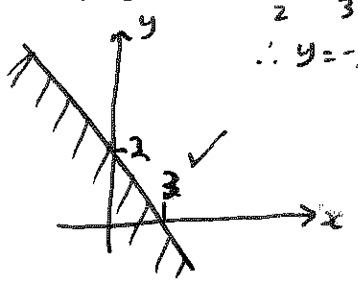
i) $x \leq -3$



ii) $y < x+3$



iii) $\frac{x}{3}+\frac{y}{2} \geq 1$



$\frac{y}{2}=\frac{-x}{3}+1$ $x=0 \Rightarrow y=2$

$\therefore y=-\frac{2x}{3}+2$ $y=0 \Rightarrow x=3$

3) A shopkeeper can buy small bottles for 8 RMB and large bottles for 24 RMB. As a new venture he is prepared to buy 80 bottles in total, of which he wants more small than large. He has up to 960 RMB to invest. He plans to sell the bottles at 12 RMB and 32 RMB respectively. Find out how many of each he should buy to maximise his profit.

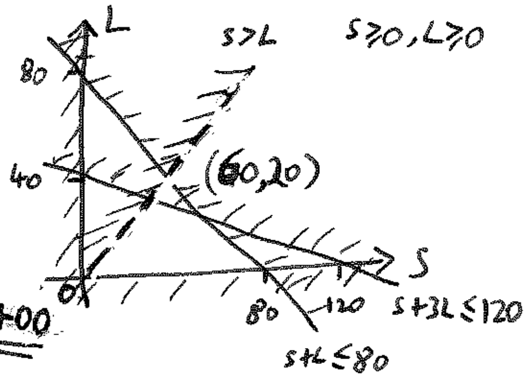
$S=8¥$, $L=24¥$ $S+L \leq 80$ ①, $S>L$ ③

$8S+24L \leq 960 \Rightarrow S+3L \leq 120$ ②

② − ① $\Rightarrow 2L=40$

So $L=20$ and $S=60$

②&③ $\Rightarrow S=L=30$



Profit, $P=4S+8L$. Consider 3 cases:

i) $(80,0) \Rightarrow P=320$ ii) $(33,29) \Rightarrow P=364$ iii) $(60,20) \Rightarrow P=400$

4) Solve the following. *Note that quadratics should normally have 2 solutions.*

a) $x(x-10)=16(1-x)$

$x^2-10x=16-16x$

$\therefore x^2+6x-16=0$

$\therefore (x+8)(x-2)=0$

So $x=-8$ or 2

b) $(x+6)^2=36$

$x+6=\pm 6$

$\therefore x=0$ or -12

c) $(2y+1)(y+1)=6(2y+1)$

$2y^2+3y+1=12y+6$

$\therefore 2y^2-9y-5=0$

$\therefore (2y+1)(y-5)=0$

So $y=-\frac{1}{2}$ or 5

5) Solve the following pair of simultaneous equations.

a) $x^2 - 3x + 4 = y$ and $y - x = 1$

$y = x+1 \quad \therefore x+1 = x^2 - 3x + 4$

$\therefore x^2 - 4x + 3 = 0$

$\therefore (x-1)(x-3) = 0$

So $x = 3$ or 1 and $y = 4$ or 2

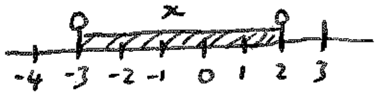
$\therefore$ Solutions are $(3,4)$ and $(1,2)$

6) Draw a number line to show for which values the following inequality is true: $6 - x - x^2 > 0$

$x^2 + x - 6 < 0$

$(x+3)(x-2) < 0$

$\therefore 2 > x > -3$



7) Find the minimum or maximum values of the following and state which it is (x and y are real numbers):

a) $x^2 + 6x - 8$

$(x+3)^2 - 9 - 8$

$\therefore$ min. at $(-3, -17)$

$\therefore$ min $= -17$

b) $6 + 6x - 2x^2$

max. at $x = 1\frac{1}{2}$

$\therefore$ max $= 11\frac{1}{2}$

c) $x^2 + 4x + 7$

min. at $x = -2$

$\therefore$ min $= 3$

d) $(y+3)^2$

min. at $y = -3$

$\therefore$ min $= 0$

8) Sketch the following graphs (labeling the vertex and x- and y- intercepts):

a) $y = x^2 + 3x + 4$

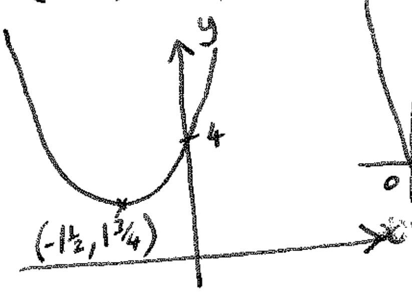
b) $y = x^2 - 6x$

c) $y = x^2 - 4x + 4$

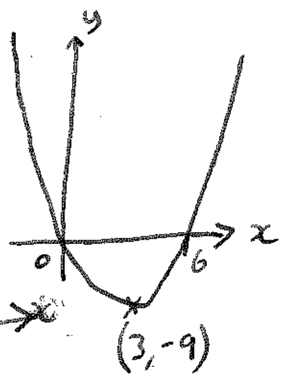
d) $y = -x^2 + 1 = 1 - x^2 = (1-x)(1+x)$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 4$

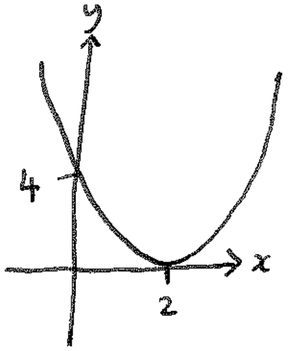
$= \left(x + \frac{3}{2}\right)^2 + 1\frac{3}{4}$

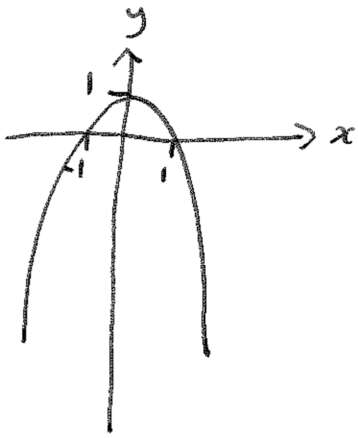


$y = x(x-6)$



$y = (x-2)^2$





9) Find the equations for the parabolas which pass through the following points:

a) (0, 3) with vertex at (1, 1)

$y = a(x-1)^2 + 1$

$(0,3) \Rightarrow 3 = a + 1$

$\therefore a = 2$

So $y = 2(x-1)^2 + 1$

$= 2x^2 - 4x + 3$

b) (2, 10) with vertex at (3, -8)

$y = a(x-3)^2 - 8$

$(2,10) \Rightarrow 10 = a - 8$

$\therefore a = 18$

So $y = 18(x-3)^2 - 8$

$= 18x^2 - 108x + 154$

b) (1, 1), (0, 4), (5, 9)

$(1,1) \Rightarrow 1 = a + b + c$ ①

$(0,4) \Rightarrow 4 = c$

$(5,9) \Rightarrow 25a + 5b + 4 = 9$ ②

$\therefore$ ① $\Rightarrow a + b = -3$

So $5a + 5b = -15$ ③

② − ③ $\Rightarrow 20a = 20$

$\therefore a = 1$

and $b = -4$

So $y = x^2 - 4x + 4$

c) (-1, 0), (3, 0), (4, 10)

$(-1,0), (3,0) \Rightarrow$ vertex at $x = 1$

$\therefore y = a(x-1)^2 + k$

$(-1,0) \Rightarrow 0 = 4a + k$

$(4,10) \Rightarrow 10 = 9a + k$

$\therefore 5a = 10$

So $a = 2$

and $k = -8$

$\therefore y = 2(x-1)^2 - 8$

$= 2x^2 - 4x - 6$